Maximal Braking on a Standard Bicycle

Version 1.0

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Introduction

The maximum steady-state braking deceleration achievable by a cyclist riding a standard bicycle is limited by pitch-over [1]. In this article an expression is derived for the maximum deceleration on a level road. It is then extended to a road with a slope. A simple method is described for measuring the position of the center of mass (CM) of a cyclist.

Level Ground

Consider a bicycle and rider (BR) braking on level ground. If the coefficient of friction between tire and road is large enough (and it generally is), the bicycle rotates about the ground contact point of the front wheel when the acceleration exceeds a maximum value, $a_{\max}$. This value is computed by assuming that the

$$\sum F_x : \; m a_{\max} = F_B, \tag{1}$$

$$\sum M|_{P_1} : \; 0 = F_B h_{\mathrm{cm}} - mg\ell_{\mathrm{f}}. \tag{2}$$

Solve for $a_{\max}$ gives

$$a_{\max} = g h_{\mathrm{cm}}/\ell_{\mathrm{f}}. \tag{3}$$

This shows that the maximum deceleration depends on the location of the CM of the BR.

Slopes

Now consider the case where the road is not level, that is has a slope s.

$$\sum F_x : m a_{\max} = F_B, \tag{4}$$

$$\sum M|_{P_1} : 0 = F_B h_{\mathrm{cm}} - mg(\ell_{\mathrm{f}} \cos\theta + h_{\mathrm{cm}} \sin\theta). \tag{5}$$

Solving for $a_{\max}$ gives

$$a_{\max}(\theta) = \left(\frac{\ell_{\mathrm{f}}}{h_{\mathrm{cm}}} \cos\theta + \sin\theta \right) g. \tag{6}$$

For slopes in the range $(-20\%, 20\%)$, $\cos\theta \approx 1$, $\sin\theta \approx \theta$, so

$$a_{\max}(s) \approx a_{\max}(0) + s g. \tag{7}$$

For example, if the maximum braking acceleration to avoid pitchover on level ground is 0.7 g, then when descending a 10% slope ($s = -0.1$) the maximum braking acceleration is $0.7\,\mathrm{g} - 0.1\,\mathrm{g} = 0.6\,\mathrm{g}$.

Locating the Center of Mass

As shown in the preceding analyses, the location of the CM of the BR significantly affects the available braking deceleration. This section describes a simple measurement technique for locating the CM of the BR.

The fractional weight, k_{wr}, on the rear tire is

$$k_{\mathrm{wr}} = 1 - \frac{W_{\mathrm{f}}}{W_{\mathrm{tot}}}, \tag{8}$$

$$= \frac{\ell_{\mathrm{f}}}{\ell_{\mathrm{wb}}} - \frac{\ell_{\mathrm{h}}}{\ell_{\mathrm{wb}}} \frac{\ell_{\mathrm{s}}/\ell_{\mathrm{wb}}}{\sqrt{1-(\ell_{\mathrm{s}}/\ell_{\mathrm{wb}})^2}}, \tag{9}$$

$$= k_{\mathrm{f}} - k_{\mathrm{h}} \frac{k_{\mathrm{s}}}{\sqrt{1-k_{\mathrm{s}}^2}}, \tag{10}$$

where W_{tot} is the total weight of the BR, W_{f} is the weight on the front wheel, ℓ_{f}, ℓ_{wb}, and ℓ_{s} are shown in figure 1, and k_{f}, k_{h}, and k_{s} are the respective lengths normalized to the wheel base, that is, $\ell_{\mathrm{f}}/\ell_{\mathrm{wb}}$, $\ell_{\mathrm{h}}/\ell_{\mathrm{wb}}$, and $\ell_{\mathrm{s}}/\ell_{\mathrm{wb}}$, respectively. To determine k_{f} and k_{h}, measure k_{wr} while varying the height of the rear axle above the front axle. This can be readily done with a bathroom scale and an assortment of bricks or blocks of wood to raise the rear or front of the bicycle.

Figure 2 plots the data for a rider on a standard (lightweight) road bike in two positions: sitting on the saddle and *hanging off* (behind) the saddle.

The parameters k_{f} and k_{h} correspond to the value and (negative) slope of k_{wr} at $k_{\mathrm{s}} = 0$, respectively. The location of the CM is given by

$$\ell_{\mathrm{f}} = \ell_{\mathrm{wb}} k_{\mathrm{f}}, \tag{11}$$

$$h_{\mathrm{cm}} = \ell_{\mathrm{wb}} k_{\mathrm{h}} + R_{\mathrm{w}}. \tag{12}$$

The wheelbase of the bicycle, ℓ_{wb}, is 100 cm, the wheel radius, R_{w}, is 34 cm. Using values of k_{f} and k_{h} extracted from the graphs in figure 2 gives

$$\begin{aligned}
\ell_{\mathrm{f(on)}} &= (100\ \mathrm{cm})(0.58) = 58\ \mathrm{cm}, \\
h_{\mathrm{cm(on)}} &= (100\ \mathrm{cm})(0.58) + 34\ \mathrm{cm} = 92\ \mathrm{cm}, \\
\ell_{\mathrm{f(behind)}} &= (100\ \mathrm{cm})(0.71) = 71\ \mathrm{cm}, \\
h_{\mathrm{cm(behind)}} &= (100\ \mathrm{cm})(0.52) + 34\ \mathrm{cm} = 86\ \mathrm{cm}.
\end{aligned}$$

The maximum braking acceleration on level ground is then

$$\begin{aligned}
a_{\mathrm{max(onsaddle)}} &= 58/92\,\mathrm{g} = 0.63\ \mathrm{g}, \\
a_{\mathrm{max(offsaddle)}} &= 71/86\,\mathrm{g} = 0.83\ \mathrm{g}.
\end{aligned}$$

References

[1] David Gordon Wilson with Jim Papadopoulos. *Bicycling Science*. Massachusetts Institute of Technology, third edition, 2004.

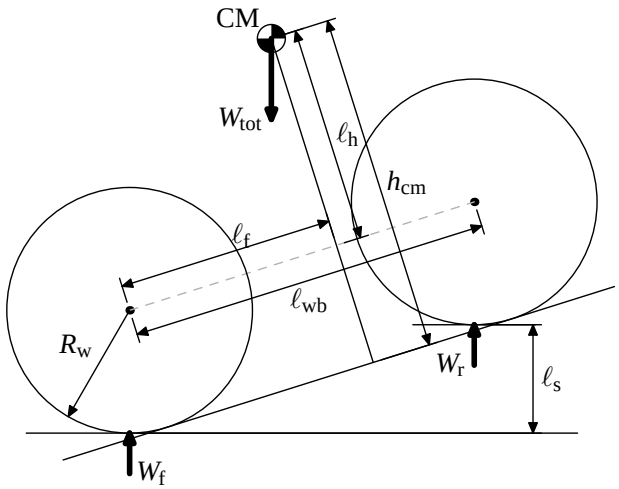

Figure 1: Locating the CM

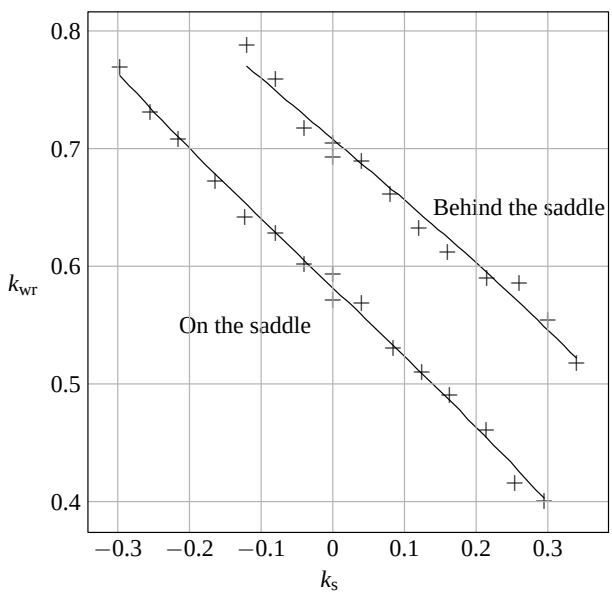

Figure 2: k_{wr} versus k_s. The lower curve is for the cyclist sitting on the saddle; the upper curve is for the cyclist behind the saddle. For both positions hands are in the drops, gripping the brake levers. Both data and fitted curves are shown for each position.