

# Evaluating the health benefit of bicycle helmet laws

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## Abstract

A model is developed which permits the quantitative evaluation of the health benefit of bicycle helmet laws. The efficacy of the law is evaluated in terms of the percentage drop in bicycling, the percentage increase in the cost of an accident when not wearing a helmet, and a quantity here called the “bicycling beta.” The approach balances the health benefits of increased safety against the health costs due to decreased cycling. Using estimates suggested in the literature of the health benefits of cycling, accident rates and reductions in cycling, suggest helmets laws are counterproductive in terms of net health. The model serves to focus the bicycle helmet law debate on overall health as function of key parameters: cycle use, accident rates, helmet protection rates, exercise and environmental benefits. Empirical estimates using US data suggests the strictly health impact of a US wide helmet law would cost around \$5 billion per annum. In the UK and The Netherlands the net health costs are estimated to be \$0.4 and \$1.9 billion, respectively.

*Key words:* Bicycling, helmets, cost benefit analysis.

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## 1. Introduction

It is generally accepted that compulsory bicycling helmet laws reduce cycling injuries and fatalities. This reduction in harm is achieved in two ways:

1. By reducing, on average, the injury cost of each cycling accident (Thompson et al., 1989).
2. By reducing the amount of cycling and hence the number of cycling accidents (Robinson, 2007, 2006).

The magnitudes of these two effects are subject to much discussion – see for example the responses in BMJ (2006) to Robinson (2006).

The disincentive effect of helmets on cycling may be partly due to the small burden of wearing a helmet, and partly due to the attention it draws – too much attention some argue (Wardlaw, 2000, 2002) – to the risks associated with bicycling. For a balanced overview of the debate see Hurst (2004, Chapter 4) or Towner et al. (2002).

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Generally there has been solid support for bicycle helmet laws in Canada, Australia and New Zealand, less so in the US and the UK, and little support in northern European countries such as the Netherlands and Denmark, where cycling is far more popular. Reasons for lack of support include the following. First, helmets reduce cycling and hence the impact of “safety in numbers.” Second, helmets may promote the view and possible misconception that cycling is a relatively dangerous activity. Third, helmets may draw attention and harm prevention activities away from the real causes of cycling accidents. Finally, helmets may promote the view that cycling is a non normal activity only to be undertaken with specialized equipment.

A reduction in cycling has negative environmental and health consequences. DeMarco (2002) opines: “Ultimately, helmet laws save a few brains but destroy many hearts.” Ignoring any environmental costs associated with reduced cycling, the efficacy of helmet laws hinges on whether the positive direct benefits – fewer head injuries – outweigh the indirect negative effects – less exercise.

Accidents, including cycling accidents and associated head injuries, are a probabilistic phenomenon requiring a probabilistic analysis. Quantifying the benefits of a helmet law requires a model that captures the important probabilistic forces. This paper develops such a model. Before proceeding some notes of clarification are in order:

- Even if an analysis suggests there is no net societal benefit of a helmet law, it may still make eminent sense for individuals to wear a helmet. Thus the analysis of this paper does not address the issue of whether a helmet in an accident confers a benefit on the wearer. Indeed the discussion below assumes that a helmet on average is beneficial in a head impact accident.
- The discussion below is in terms of averages. This smooths over accident to accident variability and in particular catastrophic events caused or avoided by bicycling – both with and without a helmet.
- No position is taken on how much on average helmets reduce the “costs” of an accident, the amount by which helmet laws reduce cycling, and the possible non health related benefits of increased cycling. However we do use widely cited estimates as input into our model to arrive at the net implied benefit. These inputs can be discussed and varied. However the model implications are not contestable.
- The analysis does not distinguish between different groups of bicycle riders. Different groups may have different accident rates – for example children or mountain bicyclists appear to have higher accident rates. Thus a targeted helmet law may make sense.

The further sections of this article are structured as follows. The next section presents a surprising but useful expression for the net health benefit of a helmet law. The expression has three key quantities: the percentage drop in cycling, the percentage increase in health costs when not wearing a helmet, and a quantity here called the bicycling “beta of bicycling.” The meaning of “beta”

is explored in §3. Section 4 goes on to consider different models for the impact of helmets on injuries and in particular head injuries. These models provide detailed insight into the likely size of percentage increase in health costs when not wearing a helmet. Section 5 goes on to consider health effects under partial uptake of helmets. This is followed in §6 by actual calculations. These calculations, corresponding to a wide variety of scenarios, suggest helmet laws are counterproductive in terms of net health benefits. Section 7 uses figures from the US, the UK and the Netherlands to compute the potential net health cost of helmet laws. The final section gives conclusions.

## 2. The net health benefit of a helmet law

Suppose  $v$  is the health benefit associated with 1 km of cycling. If there is an accident,  $C$  is the accident cost for a non-helmeted cyclist, reducing to  $C^*$  if a helmet is worn. All quantities here and below with an asterisk  $*$  indicate the values when a helmet is worn.

Hence the net benefit of riding one km is  $v$  if there is no accident and  $v - C$  if there is one accident. Suppose the rate  $\lambda$  of accidents is not affected by wearing a helmet and that  $E(C) = c$  and  $E(C^*) = c^*$ .

If  $m$  km are cycled in the absence of a helmet law and  $m^*$  if a helmet law is passed then the health benefit (HB) of helmet law is

$$\text{HB} = m^*(v - \lambda c^*) - m(v - \lambda c) \quad (1)$$

The first term on the right is the number of km cycled with a helmet law times the health benefit per km cycled. The second term on the right is the same benefit without a helmet law. In (1) it is assumed before the law nobody wears a helmet while with the law there is 100% compliance. This assumption is relaxed in §5.

A small algebraic manipulation displayed in the Appendix leads to the following surprising and useful result

$$\% \text{HB} \equiv \frac{\text{HB}}{m\lambda(c - c^*)} = 1 - \beta \frac{\Delta m}{\Delta c}, \quad (2)$$

where

$$\beta \equiv \frac{v - \lambda c^*}{\lambda c^*}, \quad \Delta c \equiv \frac{c - c^*}{c^*}, \quad \Delta m \equiv \frac{m - m^*}{m}. \quad (3)$$

The quantity  $\beta > 0$  is called the “beta of bicycling” and measures the expected net health benefit of helmeted cycling expressed as a proportion of the expected accident health cost of helmeted cycling. Further  $\Delta c$  is the proportionate increase in the expected cost of an accident when not wearing a helmet while  $\Delta m$  is the proportionate decrease in cycling. Note all three quantities are independent of measurement units.

The left hand side of (2) is HB expressed as a proportion of the total health benefit  $m\lambda(c - c^*)$  which would accrue if there were no reductions in cycling.

Hence (2) is a percentage measure of the health impact of the law. From (2) it follows

$$\text{HB} > 0 \quad \iff \quad \Delta c > \beta \Delta m . \quad (4)$$

Hence a helmet law leads to a net health benefit if and only if the expected percentage increase in the cost of an accident when not wearing helmet exceeds  $\beta$  times the percentage drop in cycling.

To illustrate, suppose  $\Delta c = 2$ , indicating a 200% increase in expected accident costs if not wearing a helmet, and  $\Delta m = 0.3$ , indicating a 30% reduction in cycling on account of the helmet law. Then the law leads to a net health benefit if  $\beta < 2/0.3 = 6.7$ . If  $\beta = 5$  then (2) equals 0.5 indicating the law only has 50% effectiveness. If  $\beta = 20$  then (2) equals -0.4 indicating the law has a net negative impact equal to 40% of the maximum positive effect. The next section puts  $\beta$  in context.

### 3. The beta of bicycling

The beta of bicycling,  $\beta$ , defined in (3) is, from (1), critical to determining whether there is a net societal health benefit of a bicycle helmet law. The numerator,  $v - \lambda c^*$ , of  $\beta$  is the expected net health benefit of cycling one km. The denominator,  $\lambda c^*$ , is the expected cost of accidents per km. Hence  $\beta$  is the (helmeted) cycling distance required to incur an expected accident cost equal to the health benefit of 1 km of riding. Put another way,  $\beta$  is the net expected health benefit when cycling as many km as required to have an expected accident cost equal to the benefit of cycling 1 km.

For example if 21 km of bicycling incurs an expected accident cost equivalent to the benefit of 1 km of cycling then  $\beta = 20$ . The literature suggests 20:1 is an appropriate figure for the benefit of unhelmeted bicycling suggesting  $\beta$  is higher than 20. For example a study by the British Medical Association (Hillman, 1992), suggest the average gain in “life years” through improved fitness from (unhelmeted) cycling exceeds the average loss in “life years” through cycling accidents by a factor of 20 to 1.

The bicycling beta  $\beta$  will vary with circumstance and individuals. For example if the risks of cycling in a given geographical area are high then  $\beta$  will be low. Inexperienced rider will have relatively low  $\beta$ 's. This suggests  $\beta$  in, say, the UK will be lower than  $\beta$  in, say, the Netherlands. This might be a basis for helmet legislation in the UK but not the Netherlands. Alternatively it might be a basis for taking measures to increase  $\beta$  in the UK by reducing  $\lambda$  or  $c^*$ .

### 4. Models for the protective effect of helmets

Helmets offer protection against head injuries but not all accidents involve heads. Accordingly this section separates injuries into head and non head injuries and analyzes the impact on the HB.

As in §2, write  $C$  as the cost of an accident and decompose  $C$  according to

$$C = I \times H + B ,$$

where  $I$  is an indicator of whether or not there is a head injury,  $H > 0$  is the cost associated with a head injury, and  $B \geq 0$  cost of a non head or “body” injury in the event of an accident. Then

$$c \equiv E(C) = E(I)E(H) + E(B) = \pi h + b ,$$

where  $\pi \equiv E(I)$  is the probability, for an unhelmeted rider, of a head injury if there is an accident,  $h \equiv E(H)$  is the expected size of a head injury if there is a head injury, and  $b \equiv E(B)$  is the expected size of a body injury in an accident.

#### 4.1. Helmets moderate the expected size of a head injury

Suppose a helmet changes  $H$  to  $H^*$ . Then

$$C^* = (I \times H^*) + B , \quad c^* \equiv E(C^*) = \pi h^* + b , \quad h^* \equiv E(H^*) .$$

and

$$\Delta c = \frac{\pi(h - h^*)}{\pi h^* + b} = \frac{\Delta h}{1 + r^*} , \quad \Delta h \equiv \frac{h - h^*}{h^*} , \quad (5)$$

where  $r^* = b/(\pi h^*)$  is the ratio of expected body injuries to expected head injuries for helmeted cyclists. Note  $\Delta h$  is the expected increase in head costs when not wearing a helmet and  $r^*$  is the rate of body to head costs when wearing a helmet. Combining (5) with (4) shows

$$\text{HB} > 0 \quad \iff \quad \frac{\Delta h}{1 + r^*} > \beta \Delta m . \quad (6)$$

Hence there is a net health benefit if and only if the “discounted” change in expected head injury costs exceeds beta times the percentage drop in cycling.

A special case is where helmets reduce all head injuries proportionally:  $H^* = (1 - \delta)H$ . Thus  $\delta$  is a measure of the effectiveness of helmets in preventing head injuries with  $\delta = 1$  corresponding to 100% effectiveness. Hence  $\Delta h = \delta/(1 - \delta)$  and a detailed calculation shows

$$\% \text{HB} = 1 - \beta \left( \frac{1}{\delta q} - 1 \right) \Delta m , \quad (7)$$

where  $q$  is the proportion of injury costs due to head injuries, for unhelmeted cyclists.

#### 4.2. Helmets moderate the probability of a head injury

In this case the rate  $\pi$  of head injuries is reduced to  $\pi^*$  say. Then  $c^* = \pi^* h + b$  and a straightforward calculation shows

$$\text{HB} > 0 \quad \iff \quad \frac{\Delta \pi}{1 + r^*} > \beta \Delta m . \quad (8)$$

where  $r^* = b/(\pi^* h)$ .

The standard statistical method for quantifying the protective effects of helmets is to estimate the change in  $\pi$  when a helmet is worn. These statistical

studies usually model the helmet effect as changing the head injury odds from  $\pi/(1-\pi)$  to  $(1-\rho)\pi/(1-\pi)$  say where  $\rho$  is the percentage reduction in the odds ratio. The size of  $\rho$  is estimated using logistic regression. With this change in the odds

$$\Delta\pi = (1-\pi)\frac{\rho}{1-\rho} \quad \Rightarrow \quad \frac{\Delta\pi}{1+r^*} = \left(\frac{1-\pi}{1+r^*}\right)\frac{\rho}{1-\rho} . \quad (9)$$

Combining the right hand side expression with (8) yields, upon rearrangement,

$$\text{HB} > 0 \quad \Leftrightarrow \quad \rho > 1 - \left(1 + \frac{1+r^*}{1-\pi}\beta\Delta m\right)^{-1} . \quad (10)$$

For example, in a much cited article on the effectiveness of bicycle helmets, Thompson et al. (1989) state “... riders with helmets had an 85% percent reduction in their risk of head injury.” This  $\rho = 0.85$  figure is widely cited by proponents of bicycle helmet usage. Others (Curnow, 2005; Robinson, 2006) have criticized the figure as either unsubstantiated or likely to be a significant overestimate. But suppose the  $\rho = 0.85$  is accurate and  $\beta = 20$ ,  $\Delta m = 20\%$ ,  $\pi = 0.5$ , and  $r^* = 30\%$ . Then from (10), there is no net health benefit to a bicycle helmet law since  $\rho$  must be greater than 91.3%.

#### 4.3. Helmets moderate the distribution of head injuries

Suppose a helmet serves to eliminate a head injury if  $H \leq d$  while for  $H > d$  a helmet reduces the cost of injury to  $H - d$ . In other words proportion  $1 - \delta$  of head injuries are reduced to zero with the remaining more severe reduced by  $d$ . Hence  $H^* = |H - d|^+$  and  $P(H > d) = \delta$ . Formulas take on a convenient if  $H$  has the Pareto distribution (Klugman et al., 1998) which is often used to model catastrophes. Then

$$P(H > x) = \left(\frac{\theta}{x + \theta}\right)^\gamma \quad \Rightarrow \quad h \equiv E(H) = \frac{\theta}{\gamma - 1}, \quad d = \theta z - 1 .$$

where  $\gamma > 1$  and  $\theta > 0$  are parameters and  $z \equiv \delta^{-1/\gamma}$ . Further

$$h^* = \frac{\theta}{\gamma - 1} \left(\frac{\theta}{d + \theta}\right)^{\gamma-1} = h\delta z \quad \Rightarrow \quad \Delta h = \frac{1 - \delta z}{\delta z} \leq \frac{1 - \delta}{\delta} .$$

and

$$\text{HB} > 0 \quad \Leftrightarrow \quad \frac{\Delta\delta}{1+r^*} > \beta\Delta m , \quad (11)$$

where  $\Delta\delta \approx (1-\delta)/\delta = \Delta\pi$  and  $r^* \approx b/(\pi\delta h) = b/(\pi^*h)$  where  $\pi^* = \delta\pi$ . Hence this case is, for large  $\gamma$ , effectively the same as modifying the probability of a head injury as described in §4.2.

A modification is to assume helmets do not moderate head injuries above threshold  $d$  in which case  $h^* = h\delta^{1-1/\gamma} + \delta d = h\delta z$  where now  $z \equiv \delta^{-1/\gamma} + d/h$ . Hence in this case the helmet effect can also be viewed as approximately changing the probability  $\pi$  of a head injury.

## 5. Effect of helmet usage rates

Some cyclist wear a helmet if there is no helmet law. And not every cyclist wears a helmet if there is a helmet law. For example Robinson (2007) reports that Australian helmet laws increased wearing rates from a pre-law average of 35% to a post-law average of 84%. Also helmets may be improper or ill fitting negating at least some of the benefits of wearing a helmet.

Suppose  $\phi$  and  $\phi^*$  are the proportions of cyclers wearing an effective helmet pre and post helmet law, respectively with  $\phi < \phi^*$ . Then in the Appendix it is shown that the leading 1 in the right hand of (2) and hence (7) is replaced by

$$\alpha \equiv (1 - \phi) - (1 - \phi^*)(1 - \Delta m) = (\phi^* - \phi) + (1 - \phi^*)\Delta m . \quad (12)$$

Thus the direct injury benefit  $\Delta c$  is scaled down by  $\alpha$ . Hence with partial helmet uptake

$$\text{HB} > 0 \quad \iff \quad \alpha\Delta c > \beta\Delta m . \quad (13)$$

In relation to (12) and (13) note:

- Since  $0 < \phi < \phi^* < 1$  and  $0 < \Delta m < 1$  it follows  $0 < \alpha < 1$ . Hence the effect of  $\phi$  and  $\phi^*$  is to make a net health benefit more unlikely.
- The terms  $1 - \phi$  and  $1 - \phi^*$  are the proportions of unhelmeted pre and post law cyclists, respectively. Further  $1 - \Delta m = m^*/m$  implying  $(1 - \phi^*)\Delta m$  is the proportion of unhelmeted cyclists who continue to cycle unhelmeted. Hence  $\alpha\Delta c$  is the fraction of  $\Delta c$  attributable to unhelmeted cyclists, net of the effect of those who remain unhelmeted.
- A helmet law, strictly enforced or otherwise, does not guarantee the wearing of proper helmets. Thus  $\phi^*$  refers to the proportion of effectively helmeted cyclists. This may be much less than 1 even with strict law enforcement. In particular  $1 - \phi^*$  is likely to be significantly greater than 0 even with 100% helmet law compliance.

## 6. Health efficiency of a helmet law

This section computes the net health benefit of a helmet law. To take a concrete example suppose the effect of a helmet is to proportionally reduce the expected head injury by  $0 < \delta < 1$  as in §4.1 . Then combining (7) and (12) yields

$$\% \text{HB} = (\phi^* - \phi) + \left\{ 1 - \phi^* - \beta \left( \frac{1}{\delta q} - 1 \right) \right\} \Delta m , \quad (14)$$

The percentage benefit (14) depends on six quantities and is graphed in the different panels of Figure 1:

1. In each panel it is assumed  $\phi = 0$  and  $\phi^* = 1$  and hence no helmets are worn pre helmet law and there is proper 100% compliance post-law.

2. Different panels correspond to the different proportions of head injuries as a proportion of all injuries in unhelmeted cycling:  $q = 25\%$ ,  $50\%$ ,  $75\%$  and  $100\%$ .
3. The effectiveness of helmets  $\delta$  is on each horizontal axis and ranges from  $40\%$  to  $100\%$ .
4. The beta of bicycling  $\beta = 10, 20, 30$  and  $40$ , corresponds to the different lines in each panel with the highest line in each panel corresponding to the lowest  $\beta$ .
5. The proportionate drop in cycling  $\Delta m$  is set at a modest  $10\%$ .

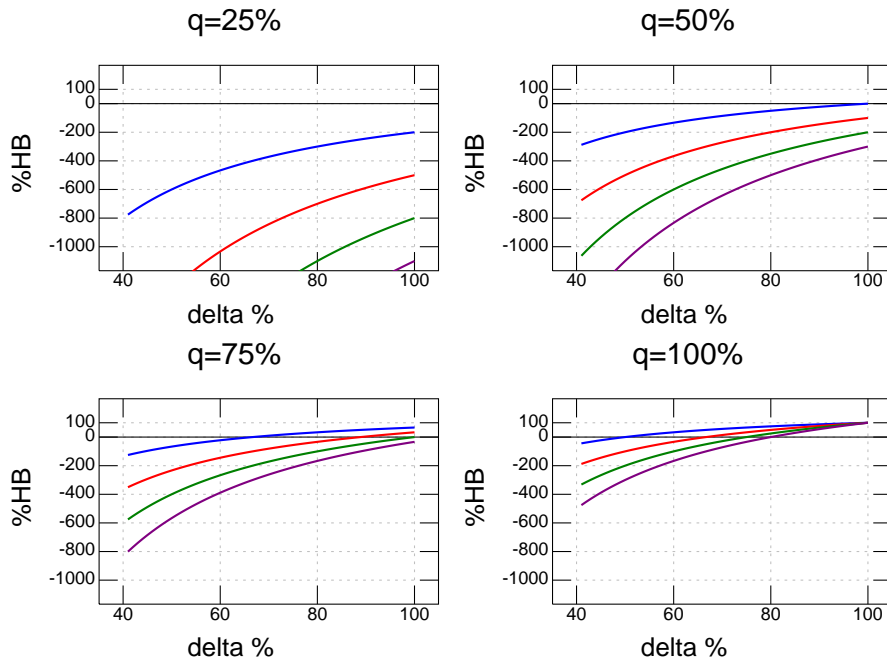


Figure 1: HB as a percentage of  $m\lambda(c - c^*)$  for  $\alpha = 1$  and  $\Delta m = 0.1$ . Different panels correspond to different  $q$ , the proportion of injury costs due to head injuries in unhelmeted cycling. Lines in each panel correspond (from highest to lowest) to  $\beta = 10, 20, 30$  and  $40$ .

Figure 1 indicates there is no positive health benefit to a helmet law except under the most optimistic assumptions as to  $q$ , the proportion of injuries that are in fact head injuries,  $\delta$ , the effectiveness of helmets,  $\Delta m$ , reductions in cycling combined with the most pessimistic assumptions about the exercise benefit of cycling. With partial uptake of helmets say  $\alpha = 50\%$  (the value implied by the Australian pre and post helmet wearing rates) a positive HB is virtually ruled out.

Even if the calculations above suggest a net health benefit other factors, ignored in the above equations, may serve to decrease the benefit:



1. Even with a strictly enforced helmet law, not everyone will wear a helmet and some may wear an ill fitting or inappropriate helmet. Hence a law may deter bicycling but may do less than expected to reduce actual head injuries.
2. The analysis assumes a helmet law does not change  $\lambda$ , the rate of accidents. Both the “risk compensation” hypothesis (Adams and Hillman, 2001) and the “safety in numbers” argument (Komanoff, 2001) would argue against this. The first states that helmets lead to an increase risk taking (moral hazard). The second states that helmets lead to increased risks for the remaining fewer cyclist. Both these effects serve to increase  $\lambda$  or  $c^*$  or both and hence a helmet law is less beneficial then indicated by the analysis of this paper.
3. It is assumed that the health benefit of accident free cycling does not change if a helmet is worn. Some authors point to inconvenience, cost and minor irritation and hence the health benefit is less. Again this would serve to decrease net benefits of a helmet law as suggested by the analysis in this article.
4. Critics of helmet laws have pointed out that by focusing on  $\Delta c$ , governments have done little to increase both the actual value  $\beta$  (through say reducing  $\lambda$  via better road design or traffic control) and the perceived  $\beta$  (through stressing the net benefits of bicycling). Emphasizing the risks of bicycling without a helmet, may lead to the mistaken perception that  $\beta$  is very low and hence increase  $\Delta m$ .
5. The above analysis only considers exercise or health benefits of cycling. There are other benefits including transportation – achieved with little pollution, no oil, comparatively minor road requirements, and so on. A bicyclists, on average, poses small risks to others, especially when compared to say a car driver. This suggests the benefit of 1 km of cycling, denominated as  $v$  in the above analysis, should be replaced by say some multiple of  $v$  to reflect these additional benefits. If so the beta of bicycling will substantially increase.
6. Helmets cost money serving to increase  $c^*$ . Many would regard the increased cost as trivial. When  $c^*$  is increased both  $\Delta c$  and  $\beta$  decrease. The cost of helmets may be better spent on other risk reducing measures (Taylor and Scuffham, 2002).

## 7. Evaluative discussion

To put the net health costs into perspective consider the US case. Hurst (2004) reports a cycling death rate of  $0.26 \times 10^{-6}$  per hour of cycling. In the US there are about 750 cycling deaths annually implying  $10^6 \times 750/0.26$  hours of annual cycling. Assuming cyclists average 10 km/h yields  $m = 10^7 \times 750/0.26 = 2.9 \times 10^{10}$  km of annual cycling. Given  $\alpha = 1$ ,  $\beta = 20$ ,  $\Delta m = 0.2$ ,  $q = 50\%$  and  $\delta = 0.30$  yields  $HB = -0.165mv$ . If  $\phi = 0.35$  and  $\phi^* = 0.84$  then with the same values of the other parameters,  $HB = -0.177mv$ . Ignoring this minor downward

adjustment, the net total annual health cost of a helmet law is equivalent to the net health benefit of  $0.165 \times 2.9 \times 10^{10} = 4.75 \times 10^9$  km of cycling. Valuing the health benefit of 1 km of cycling as  $v = 1$  dollars implies a net annual health cost of \$4.75 bn. In relation to this figure note the following:

- The cited US figures imply a fatality rate of  $2.6/10^8$ km. This compares to the UK and The Netherlands of 6.0 and 1.6 per  $10^8$  km, respectively. One would expect the US figure to be much closer to that of the UK. On the basis of the same  $\beta$  and other parameters the net annual health cost of a helmet law in the UK and The Netherlands are \$0.4 and \$1.9 bn, respectively.
- The health benefit of a km of cycling is valued at \$1. If higher or lower the above \$4.75 bn is scaled accordingly.
- The cost ignores all other costs including the cost of alternative transport – usually cars. If all “lost” cycling is replaced by car transportation then  $m\Delta m = 2.9 \times 10^{10} \times 0.2 = 5.8 \times 10^9$  additional km are traveled by cars. Valuing this at say \$1/km yields an additional cost of \$5.8 bn.
- The net health cost is based on  $\alpha = 1$  and the stated values of  $\beta$  and other parameters. These values are generally favorable to the pro-helmet case.

Hence it appears safe to conclude that helmet laws do not deliver a net societal health benefit and indeed impose a considerable health cost on society.

## 8. Conclusions

This article displays and discusses a model for evaluating the net health benefit of a bicycle helmet law. The model recognizes one health benefit – exercise – and one health cost – injuries. A positive net benefit occurs if and only if the proportionate drop in cycling multiplied by a coefficient, called the bicycling beta, is less than the proportionate increase in accident costs when not wearing a helmet. The bicycling beta captures the relative benefit of exercise compared to accidents. Using widely cited estimates of the exercise benefit of cycling, costs of head injuries and reductions in cycling leads to the conclusion that bicycle helmet laws do not deliver a positive societal health benefit. The model can be used with other estimates and casts the bicycle helmet law controversy in terms of appropriate and easily interpretable constructs.

## Appendix

This appendix proves the relation (2) and its generalization (12). To begin note that the left hand side of (2) equals

$$m^*(v - \lambda c^*) - m(v - \lambda c) - m\lambda c^* + m\lambda c^*$$

$$= (m^* - m)(v - \lambda c^*) + m\lambda(c - c^*) = mv \left\{ \frac{\Delta m(v - \lambda c^*) + \lambda c^* \Delta c}{v} \right\},$$

which simplifies to the right hand side of (2).

To prove (12), the net health benefit of a helmet law under the stated conditions is

$$\begin{aligned} & m^*[v - \lambda\{c^* + (1 - \phi^*)(c - c^*)\}] - m[v - \lambda\{c^* + (1 - \phi)(c - c^*)\}] \\ &= (m^* - m)(v - \lambda c^*) - \lambda(c - c^*)\{m^*(1 - \phi^*) - m(1 - \phi)\}. \end{aligned}$$

The first term in the last expression equals (2) minus  $m\lambda(c - c^*)$ . Hence the whole expression is (2) minus

$$\begin{aligned} & m\lambda(c - c^*) + \lambda\{m^*(1 - \phi^*) - m(1 - \phi)\}(c - c^*) \\ &= m\lambda(c - c^*)\{1 + (1 - \phi^*)(1 - \Delta m) - (1 - \phi)\}. \\ &= mv \left( \frac{\lambda c^*}{v} \right) \Delta c \{1 - (\phi^* - \phi) - (1 - \phi^*)\Delta m\}, \end{aligned}$$

which yields (12).

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